20. D. L. Lamberson, J. R. Asay, and A. H. Guenther, "Equation of state of polystyrene and polymethylmethacrylate from ultrasonic measurements at moderate pressures," J. Appl. Phys., 43, No. 3 (1972).
21. Yu. V. Bat'kov and E. D. Vishnevetskii, "Instrumentation for the measurement of pulse pressures in the 0.1-20 GPa range with piezoresistive detectors," in: Abstracts of Lectures of the Second All-Union Symposium on Pulse Pressures [in Russian], VNIIFTRI, Moscow (1976).
22. V. M. Kataeva, V. A. Popova, and B. I. Sazhina (editors), Handbook on Bulk Plastics [in Russian], Vols. 1 and 2, Khimiya, Moscow (1975).

# EFFECT OF STRAIN RATE ON THE CHARACTERISTICS OF ELASTOPLASTIC DEFORMATION OF METALLIC MATERIALS

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One of the main problems of the mechanical testing of materials is the determination of their strength and deformation characteristics as a function of the temperature and time conditions of loading. For a fixed temperature and strain rate the strength of the material under investigation is determined by its structural state (conventionally denoted by C), which changes during deformation as a result of the combined action of processes related to the growth of plastic deformation and processes developing with them. Their effect on the shear strength $\tau$ can be estimated by the strain-hardening modulus $M_D = \partial\tau/\partial e_p$ $(\partial\tau/\partial t = 0)$ and the strain softening modulus $R = -\partial\tau/\partial t$ $(\partial\tau/\partial e_p = 0)$ [1], so that for a fixed plastic shear rate $\dot{e}_p$

$$\tau_{\dot{e}_p=\text{const}} = \tau(C_0, \dot{e}_p) + \int_{e_p^0}^{e_p} \left[M_D - \frac{R}{\dot{e}_p}\right] de_p \tag{1}$$

where the subscript zero denotes the initial state.

A change of the strain rate for a fixed structure of the material leads to a change of the ductile component of the strength. Characterizing the effect of rate by the viscosity factor $\mu$ — the proportionality factor between increments of rate and the ductile component of the strength — we find

$$\tau_{C=\text{const}} = \tau(C, \dot{e}_p^0) + \int_{\dot{e}_p^0}^{\dot{e}_p} \mu(C)\, d\dot{e}_p. \tag{2}$$

By taking account of Eqs. (1) and (2) we obtain the dependence of the strength on the history of previous deformation $e_p$ in the form

$$\tau = \tau(C_0, \dot{e}_p^0) + \int_0^t \left[M_D \frac{de_p(t)}{dt} - R\right] dt + \int_{\dot{e}_p^0}^{\dot{e}_p} \mu(C)\, d\dot{e}_p.$$

The results of experiments with a combined loading regime are commonly processes by using integral equations of nonlinear hereditary viscoplasticity, e.g., of the form [2]

$$\tau = \varphi(e_p) - \int_{(t)} Q_1(t-\zeta)\,\tau(\zeta)\, d\zeta \tag{3}$$

In this procedure the effect of rate on the ductile component of the strength is not separated from its effect on the change of the structural state, since the strain-hardening and relaxation processes interact. This approach encounters serious difficulties in describing

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a deformation process with a stepwise change in strain rate during which the structure of the material is practically unchanged, so that the change in strength is related only to the change of its ductile component. Therefore it is more convenient to separate out the ductile component of the strength and to describe the change in strength related to the change of structure of the material by the integral equation $\int_{(t)} [M_D \dot{e}_p(t) - \bar{R}]\, dt = \int_{(t)} Q(t-\zeta)\, \tau(\zeta)\, d\zeta$.

In this case the strain equation for the material (3) is transformed into

$$\tau = \tau(e_p, \dot{e}_p^0) + \int_{(\dot{e}_p^0)}^{\dot{e}_p} \mu d\dot{e}_p, \qquad (4)$$
$$\tau(e_p, \dot{e}_p^0) = \tau(e_p^0, \dot{e}_p^0) + \int_{(t)} Q(t-\zeta)\, \tau(\zeta)\, d\zeta,$$

where $\tau(e_p, \dot{e}_p^0)$ is the strain curve for a steady strain rate $\dot{e}_p^0$ (minimum for the range of loading conditions investigated).

At high strain rates processes related to the effect of time on the structure of the material are unimportant, and the strength is determined only by strain hardening and the ductile component

$$\tau = \tau(e_p^0, \dot{e}_p^0) + \int_{(e_p)} M_D de_p + \int_{(\dot{e}_p)} \mu d\dot{e}_p. \qquad (5)$$

Using the representation of the limiting strain curve as the curve corresponding to a zero effect of relaxation processes and zero ductile component of the strength, $\varphi(e_p) = \tau(e_p^0, \dot{e}_p^0) + \int_{(e_p)} M_D de_p$, we obtain from (4)

$$\tau = \varphi(e_p) - \int_{(t)} J(t-\zeta)\, \tau(\zeta)\, d\zeta + \int_{(\dot{e}_p)} \mu d\dot{e}_p. \qquad (6)$$

A strain equation of the form (5) in general form represents a relation $\tau = \tau(e_p, \dot{e}_p)$, which corresponds to the behavior of a material which is insensitive to the history of previous loading. Equation (4) or (6) takes account of the effect of the previous loading history on the strain curve of the material. Performing tests at a constant strain rate ensures obtaining data on the effect of rate on the strength characteristics related to a change of structure of the material during deformation at a constant rate and with the change of the ductile component of the strength. Only for high strain rates can these results be considered unrelated to a given strain law.

Experiments at rates $\dot{e}_p < 10^2$-$10^3$ sec$^{-1}$ show that the strength is a piecewise linear function of the logarithm of the strain rate $\tau = \tau_1 + K(e_p)\ln(\dot{e}_p/\dot{e}_1)$, where $\tau_1$ is the strength for a strain rate $\dot{e}_1$ and $K(e_p)$ the dynamic response factor $K = \partial\tau/\partial \ln \dot{e}_p$. Figure 1 shows the results of tests of samples of titanium alloys with a test length 4 mm in diameter and 10 mm in length (points 1 are the breaking points of the material as received, and points 2 in the thermally hardened state).

For high strain rates ($\dot{e}_p > 10^3$ sec$^{-1}$) the strength is a linear function of the strain rate [3, 4]. According to the dislocation model of the plastic flow of metals this character of the dependence of strength on the strain rate is related to the thermally activated release of dislocations from pinning at points with a potential barrier of approximately one level (for $\dot{e}_p < 10^3$ sec$^{-1}$) and with an athermal motion of dislocations at high strain rates.

The presence of several parts of a linear dependence of the strength on the logarithm of the strain rate may be related to a different type of barrier at pinning points of the dislocations, controlling their motion over a finite range of strain rates. As a consequence of the possible effect of the previous loading history noted above, the viscosity factor $\mu_C = (\partial\tau/\partial\dot{e}_p)_{C=\mathrm{const}}$ is not a proportionality factor between incremenets of strength and strain rate in such tests except at high strain rates ($\dot{e}_p > 10^3$ sec$^{-1}$) or at temperatures below the recrystallization temperature. Therefore, from the results of quasistatic

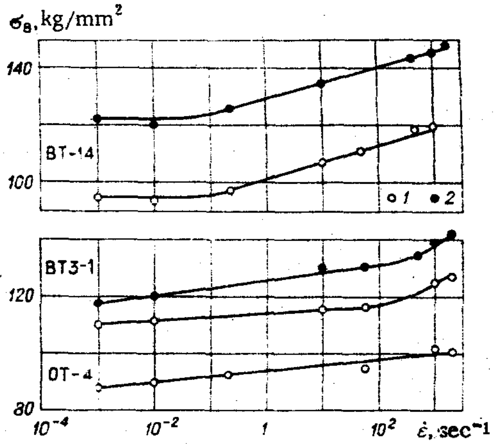


Fig. 1

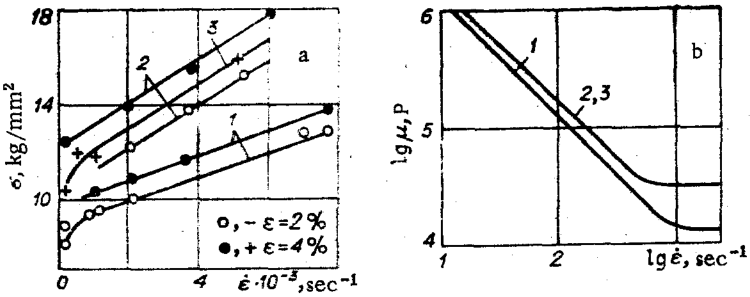


Fig. 2

tests the effect of the strain rate on the strength of metals is related to the increase of the ductile component of the strength, determined by the viscosity factor $\mu_C = (\partial\tau/\partial\dot{e}_p)_{C=\text{const}}$ and the effect of relaxation processes. The "apparent" viscosity factor $\mu = (\partial\tau/\partial\dot{e}_p)_{e_p=\text{const}}$ is a fundamental characteristic, determined in experiments at various strain rates and characterizing the sensitivity of the material to the strain rate.

While experiments at strain rates below $10^2$ $\text{sec}^{-1}$ present no difficulties and can be performed by quasistatic tests (a uniform state of stress and strain is maintained in the test length of the sample during the test, as in static experiments), measurements at higher strain rates require special methods. The effects of longitudinal and radial inertia, and wave processes in the sample and other elements of the loading equipment, make it impossible to maintain a specified state of stress in the test length of the sample, and this lowers the accuracy of the determination of the stresses; therefore, the most acceptable test at high strain rates is one at a constant strain rate (accelerations and forces connected with inertial effects are unimportant for such a loading law). The procedures in such tests for elongation at a rate up to $4.5\cdot10^4$ $\text{sec}^{-1}$ and for compression at a rate up to $2.5\cdot10^5$ $\text{sec}^{-1}$ are presented in [5, 6]. As the strain rate is increased, the factor $\mu = \partial\tau/\partial\dot{e}_p$ determined from the results of quasistatic tests decreases to a minimum at a rate of $\sim10^3$ $\text{sec}^{-1}$.

Figure 2 shows the results of processing the experimental data of [7] for test temperatures of 295, 194, and 77.4°K (curves 1-3, respectively). This dependence for strain rates above $10^5$ $\text{sec}^{-1}$ cannot be determined from the results of quasistatic tests, because the strain in the test length of the sample is not uniform.

The most reliable way to study the characteristics of a high strain rate is to analyze the laws of propagation of longitudinal elastoplastic loading waves. The possibility of not taking account of the effect of the loading history permits an analysis with an equation of state of the form $F(\tau, e_p, \dot{e}_p) = 0$. The highest strain rate occurs at the front of a plastic wave, but the fact that the rheological behavior of the material is manifested only in a change of the wave velocity complicates the determination of the parameters which characterize the behavior of the material at a high strain rate. Therefore, an analysis of data in the literature is limited to an estimate of the dynamic yield stress from the ampli-

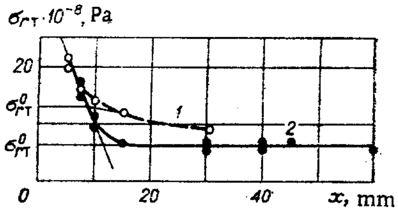


Fig. 3

TABLE 1

| Material | $\nu$ | $E\cdot 10^{-4}$, kg/mm² | $G\cdot 10^{-4}$, kg/mm² | x, mm | $\sigma^0_{rT}(\sigma^0_T)$, kg/mm² | $\sigma_{rT}(\sigma_T)$, kg/mm² | $\zeta$, μsec | $\mu\cdot 10^{-4}$, P | $\dot{e}_p$ sec$^{-1}$ |
|---|---|---|---|---|---|---|---|---|---|
| V95 Alloy | 0,31 | 0,74 | 0,27 | 7 | 70(38,5) | 160(88) | 0,063 | 1,7 | $1{,}4\cdot 10^5$ |
| St. 20 | 0,29 | 2,10 | 0,81 | 5 | 90(53,0) | 210(124) | 0,047 | 3,8 | $4{,}4\cdot 10^4$ |
| Aluminum* | 0,31 | | 0,27 | 1,7 | 6,5(3,6) | 9,0(5,0) | 0,015 | 0,4 | $1{,}1\cdot 10^4$ |

*Calculated from experimental results reported in [9].

tude of the elastic precursor. Analytic solutions for the decay of the amplitude of the elastic precursor of a plane wave in a material with a constant viscosity factor and strain-hardening modulus can be used to find the viscosity factor for metallic materials for very high strain rates. The procedure for estimating the viscosity factor and the plastic deformation rate from the experimental decay curve of the amplitude of the elastic precursor is presented below.

For a material with a constant viscosity factor $\mu = (\partial\tau/\partial\dot{e}_p)_{e_p}$ and a constant strain-hardening modulus $M = \partial\tau/\partial e_p$, the stresses and strains during the propagation of a wave produced by the plane impact of plates of the same material with a velocity $v_0$ are determined by [8]

$$\frac{\sigma_r - \sigma^0_{rT}}{\sigma^0_{rT}} \approx H(t - x/a_0)\, v_p (B + cA); \tag{7}$$

$$\frac{\varepsilon_r - \varepsilon^0_{rT}}{\varepsilon^0_{rT}} \approx H(t - x/a_0)\, v_p (B + A), \quad v_p = \frac{v_0}{2a_0\varepsilon^0_{rT}} - 1, \tag{8}$$

$$B = \exp\left(-\frac{1+c}{2\zeta}t + \frac{cx}{a_0\zeta}\right) I_0\left(\frac{1-c}{2\zeta}\sqrt{t^2 - x^2/a_0^2}\right), \quad A = \int\limits_{x/(a_0\zeta)}^{t/\zeta} B\,dt,$$

where $\sigma^0_{rT}$ and $\varepsilon^0_{rT}$ are respectively the static yield stress and the corresponding strain for a plane wave; K and G are the bulk and shear moduli, respectively; $I_0$ is the Bessel function of imaginary argument; $c = \left(K + \frac{4}{3}M\right)/\left(K + \frac{4}{3}G\right)$; $\zeta = \frac{\mu}{G}$; $a_0 = \sqrt{\left(K + \frac{4}{3}G\right)/\rho_0}$; $H(\varphi)$ is the Heaviside unit function.

The plastic strain rate at the front of the elastic precursor of a plane wave ($\dot{e}_p = \dot{\varepsilon}_r - \dot{\tau}/G$) is determined by using the Laplace transform

$$\zeta\dot{e}_p(p, x) = \frac{p}{p+1}\left(1 - \frac{M}{G}\right)\bar{\varepsilon}_r(p, x), \tag{9}$$

$$\dot{e}_p(t, x) \approx H(t - x/a_0)\, v_p \frac{\varepsilon^0_{rT}}{\zeta}\left(1 - \frac{M}{G}\right) B,$$

obtained by taking account of the dependence of the shear strength $\bar{\tau} = \tau - \tau_T$ on the strain $\bar{\varepsilon}_r = \varepsilon_r - \varepsilon^0_{rT}$ for stresses greater than the static yield stress $\bar{\tau}(p, x) = \frac{p + M/G}{p+1}\bar{\varepsilon}_r(p, x) G$.

From Eqs. (7)-(9) we find the stress, strain, and plastic strain rate at and near the front of the elastic precursor ($x = a_0 t$):

TABLE 2

| Experimental method | Material | $\dot{e}_p$, sec$^{-1}$ | μ, P | Source of experimental data |
|---|---|---|---|---|
| I | Aluminum alloy | To $6\cdot10^3$ | $3\cdot10^5$ | [10] |
| | St. 6 | To $6\cdot10^3$ | $4\cdot10^5$ | |
| II | Niobium | | $2{,}2\cdot10^6$ | [11] |
| | St. 3 | | $3{,}9$—$4{,}8\cdot10^6$ | |
| III | Armco iron | $>10^3$ | $2{,}5\cdot10^4$ | [5] |
| | Soft steel | $>10^3$ | $2{,}1\cdot10^4$ | [4] |
| | St. 20 | $10^4$—$5\cdot10^4$ | $2{,}1\cdot10^4$ | [15] |
| | St. 45 | $>10^3$ | $2{,}3\cdot10^4$ | [5] |
| | Aluminum | $>10^3$ | $1{,}3\cdot10^4$ | [8] |
| | D16 alloy | $10^4$—$5\cdot10^4$ | $1{,}5\cdot10^4$ | [15] |
| | | $10^4$—$1{,}5\cdot10^5$ | $1{,}4\cdot10^4$ | [6] |
| IV | A19 alloy | $4\cdot10^5$—$8\cdot10^6$ | $2\cdot10^4$ | [13] |
| | Lead | $\sim10^7$ | $3{,}7\cdot10^4$ | |
| | Mercury | | $0{,}8\cdot10^4$ | [12] |
| | Water | | $2{,}2\cdot10^4$ | |
| V | St. 20 | $10^4$—$10^5$ | $3{,}2\cdot10^4$ | From rate dependence of breaking strength [14] |
| | V95 alloy | $10^4$—$10^5$ | $0{,}5\cdot10^4$ | |
| | Aluminum alloy | $10^4$—$10^5$ | $3\cdot10^4$ | |
| VI | V95 alloy | $1{,}4\cdot10^5$ | $1{,}7\cdot10^4$ | From decay of elastic precursor (see Table 1) |
| | St. 20 | $4{,}4\cdot10^4$ | $3{,}8\cdot10^4$ | |
| | Aluminum | $1{,}1\cdot10^4$ | $0{,}4\cdot10^4$ | |

$$\sigma_{rT}-\sigma^0_{rT}\approx H(t-x/a_0)\,\rho_0 a_0\left(\frac{v_0}{2}-\frac{\sigma^0_{rT}}{\rho_0 a_0}\right)\exp\left(-\frac{1-c}{2\zeta a_0}x\right),\qquad(10)$$

$$\varepsilon_{rT}-\varepsilon^0_{rT}\approx H(t-x/a_0)\left(\frac{v_0}{2}-\frac{\sigma^0_{rT}}{\rho_0 a_0}\right)\exp\left(-\frac{1-c}{2\zeta a_0}x\right)\Big/a_0,$$

$$\dot{e}_p\approx H\left(t-\frac{x}{a_0}\right)\left(\frac{v_0}{2}-\frac{\sigma^0_{rT}}{\rho_0 a_0}\right)\exp\left(-\frac{1-c}{2\zeta a_0}x\right)\Big/(a_0\zeta).$$

Equations (10) are valid when M and μ are constant, but for actual materials they vary, as indicated above, with the plastic components of the strain and strain rate. Assuming that at each instant the decay of the precursor is determined by the stress intensity and viscosity factors at that instant, the viscosity factor can be found

$$\mu=G\zeta=\frac{2}{3}G\frac{(G-M)}{\left(K+\frac{4}{3}G\right)a_0}\,\frac{\sigma_{rT}-\sigma^0_{rT}}{\left(\frac{\partial\sigma_{rT}}{\partial x}\right)}.\qquad(11)$$

At the front of the elastic precursor the plastic strain rate is related to the amplitude of the precursor:

$$\dot{e}_p=(\sigma_{rT}-\sigma^0_{rT})\left(1-\frac{M}{G}\right)\Big/(\rho_0 a_0^2\zeta).\qquad(12)$$

Using Eq. (11) for the viscosity factor, we find from (12)

$$\dot{e}_p=\frac{3}{2}\frac{a_0}{G}\frac{\partial\sigma_{rT}}{\partial x}.\qquad(13)$$

It is of interest to estimate the strain rate at the front of the elastic precursor in metals. From calculations with a relaxation time of 0.05 μsec and $v_0 - v_T$ = 200 m/sec, the plastic shear rate is $4.5\cdot10^3$ and $3\cdot10^2$ sec$^{-1}$ at distances of 5 and 10 mm from the loaded surface, respectively. This estimate confirms the previous conclusion that high strain-rate processes occur only near the loaded surface (it was assumed that M = 0 and $\sigma^0_{rT}$ is the point of intersection of the two parts of the curves in Fig. 3).

By using Eq. (13) the yield stress calculated from the amplitude of the elastic precursor $\sigma_T = \sigma_{rT}((1 - 2\nu)/(1 - \nu))$, where $\nu$ is Poisson's ratio, can be determined as a function of the rate, and compared with the results of quasistatic tests in general for their range of rates. The error in determining the viscosity factor and the strain rate comes mainly from neglecting the strain-hardening modulus, whose magnitude is unknown for high strain rates.

The calculated values of the yield stresses and viscosity factors of St. 20 steel and V95 aluminum alloy for high strain rates determined from the decay of the amplitude of the elastic precursor (curves 1 and 2 of Fig. 3) are listed in Table 1. The plastic strain rate and the viscosity factor were determined from the slope of the portions of the curve for $x < 10$ mm. The mass velocity at the wave front is 200 m/sec.

Table 2 lists the values of the viscosity factors determined by various methods at T = 20°C. The values of the factors obtained from quasistatic tests III, from the rate dependence of the breaking strength V, and from the decay of the elastic precursor VI are nearly the same. The results obtained by identical methods are listed in groups of rows I-VI.

## LITERATURE CITED


1. G. V. Stepanov, "Model of the mechanical behavior of metallic materials under pulsed loading," Prob. Prochn., No. 5 (1978).
2. Yu. N. Rabotnov, "The theory of creep," in: Fifty Years of Mechanics in the USSR, Vol. 3 [in Russian], Nauka, Moscow (1972).
3. J. Weertman, "High velocity dislocations," in: Response of Metals to High-Velocity Deformation. Interscience, New York (1961).
4. J. D. Campbell and W. G. Ferguson, "The temperature and strain-rate dependence of the shear strength of mild steel," Philos. Mag., 21, 63 (1970).
5. G. V. Stepanov, Elastoplastic Deformation of Materials under Pulsed Loading [in Russian], Naukova Dumka, Kiev (1979).
6. G. V. Stepanov and V. V. Astandin, "Test of metals under high-velocity shock compression," Prob. Prochn., No. 2 (1980).
7. F. E. Hauser, "Techniques for measuring stress—strain relations at high strain rates," Exp. Mech., 8, 395 (1966).
8. G. V. Stepanov, "Propagation of plane waves in a viscoplastic material with linear hardening," Prob. Prochn., No. 8 (1975).
9. T. E. Arvidsson, Y. M. Gupta, and G. E. Duvall, "Precursor decay of 1060 aluminum," J. Appl. Phys., 46, 4474 (1975).
10. S. M. Popov, "Absolute viscosity of steel," Inzh. Sb., No. 1 (1941).
11. I. D. Zakharenko and V. I. Mali, "Viscosity of metals under explosion welding," in: Combustion and Explosion [in Russian], Nauka, Moscow (1972).
12. V. N. Mineev and R. M. Zaidel', "Viscosity of water and mercury under shock loading," Zh. Eksp. Teor. Fiz., 54, No. 6 (1968).
13. V. N. Mineev and E. V. Savinov, "Viscosity and melting point of aluminum, lead, and sodium chloride under shock compression," Zh. Eksp. Teor. Fiz., 52, No. 3 (1967).
14. G. V. Stepanov, "Relation between resistance to deformation under uniaxial stress and breaking point for metals," Prob. Prochn., No. 12 (1979).
15. G. V. Stepanov, "High-velocity tensile test to sheet metals," Prob. Prochn., No. 7 (1980).